Pre-Acceleration Linac in ICOOL
Optimized FFAG Designs

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Outline: Pre-Acceleration Linac in ICOOL

- Modifications to ICOOL
- Longitudinal acceptance
- Lattice functions
- Future work

Modifications to ICOOL

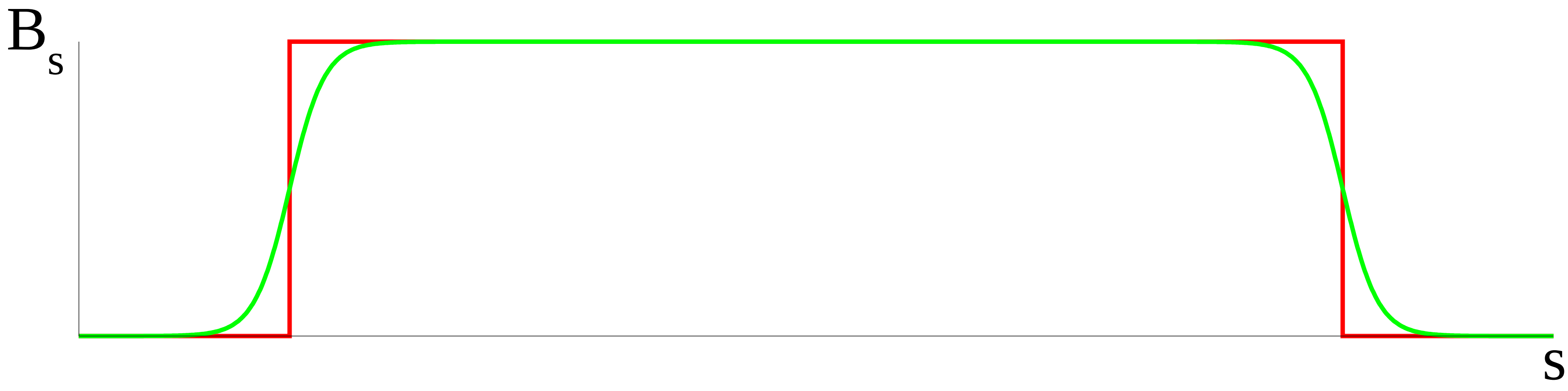
- RF phasing: new phasemodel (2)
  - Really track reference particle to define phase
    - ⋆ Zero crossing is the phase for which the reference particle gains no energy
    - ⋆ Add the cavity's phase to that, integrate reference particle through fields to get energy gain
  - Only works in restricted circumstances
    - ⋆ Assumes reference particle is on axis
    - ⋆ Only works for a couple of accel models
    - ⋆ Only works for fixed step size
    - ⋆ Code can be added to fix all these…
- New ACCEL model (13): open hard-edge pillbox cavity
  - Constant longitudinal profile, sinusoidal
  - Has hard-edge focusing on ends (can be turned off for either end)

Modifications to ICOOL (cont.)

- New SOL model (8): hard-edge solenoid
  - Simple fields: runs fast (10K particles in a few minutes)!
  - B_s constant, delta-function B_r on ends (can be selectively turned off)
  - Extra radially symmetric defocusing on ends
    - ⋆ Focusing strength is proportional to B_s^2

$$L \int_0^L B_s^2 ds < \left(\int_0^L |B_s| \, ds \right)^2$$

    - ⋆ Difference concentrated on ends: approximate by thin lens



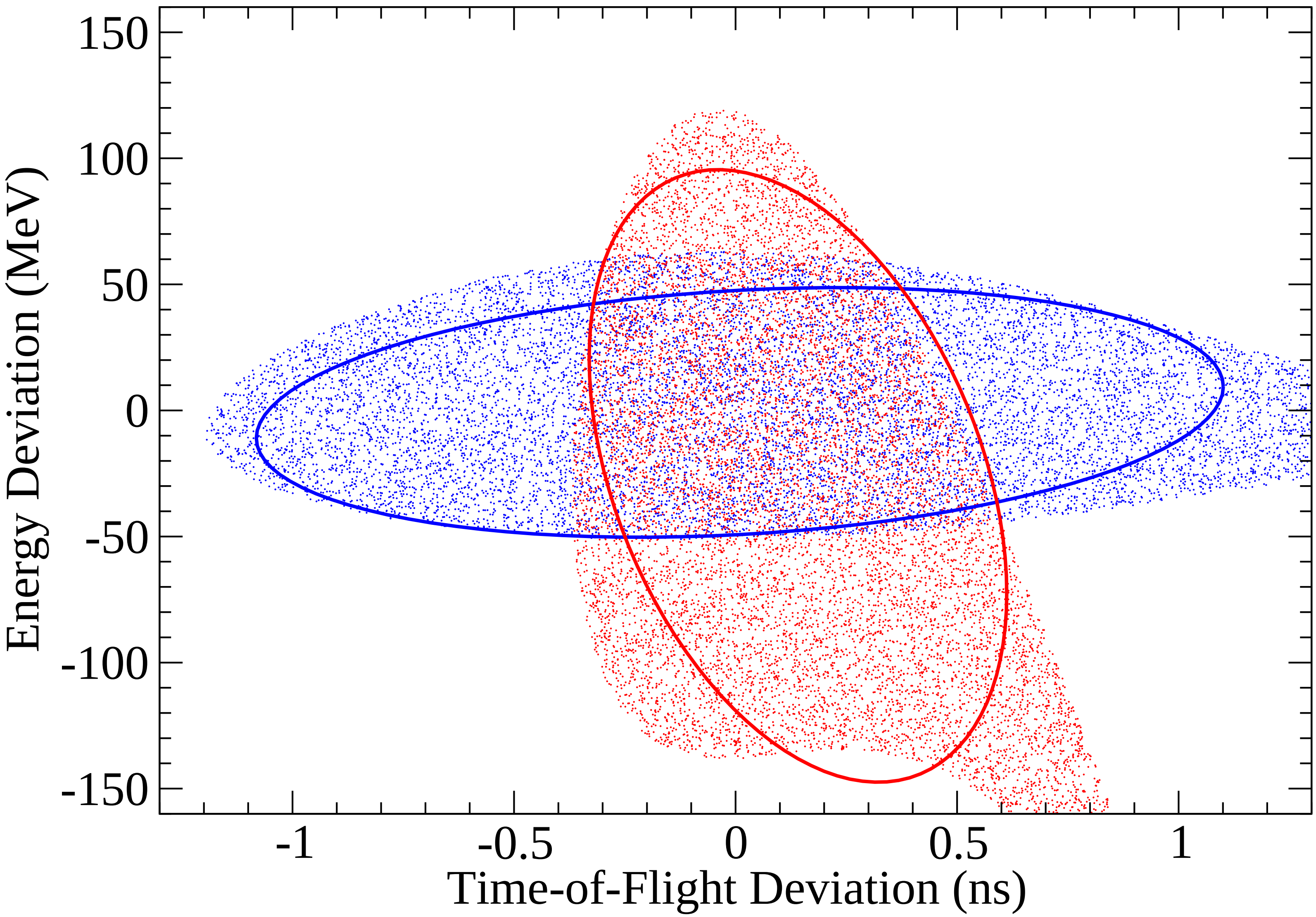
General Comments on ICOOL Run

- Matches Alex’s computation, with one exception: I use finite length cavities
  - Lower transit time factor at low energy
  - Slightly lower energy gain
  - Will have other effects (see soon)

Longitudinal Acceptance

- Start with a wide uniform longitudinal distribution
- Track to end, keeping only particles within 1/2 bucket of ref particle
- Plot: particles that make it to end (red), same particles at beginning (blue)
- Ellipses: 150 mm acceptance, orientation computed through cuts
  - Start at end
    - Compute covariance matrix
    - Remove particles outside 2.1σ
    - Repeat until no more particles cut
  - Remove corresponding particles at beginning, do iterative cut on beginning distribution
  - Remove corresponding particles at end, compute covariance matrix and draw ellipses

Longitudinal Acceptance



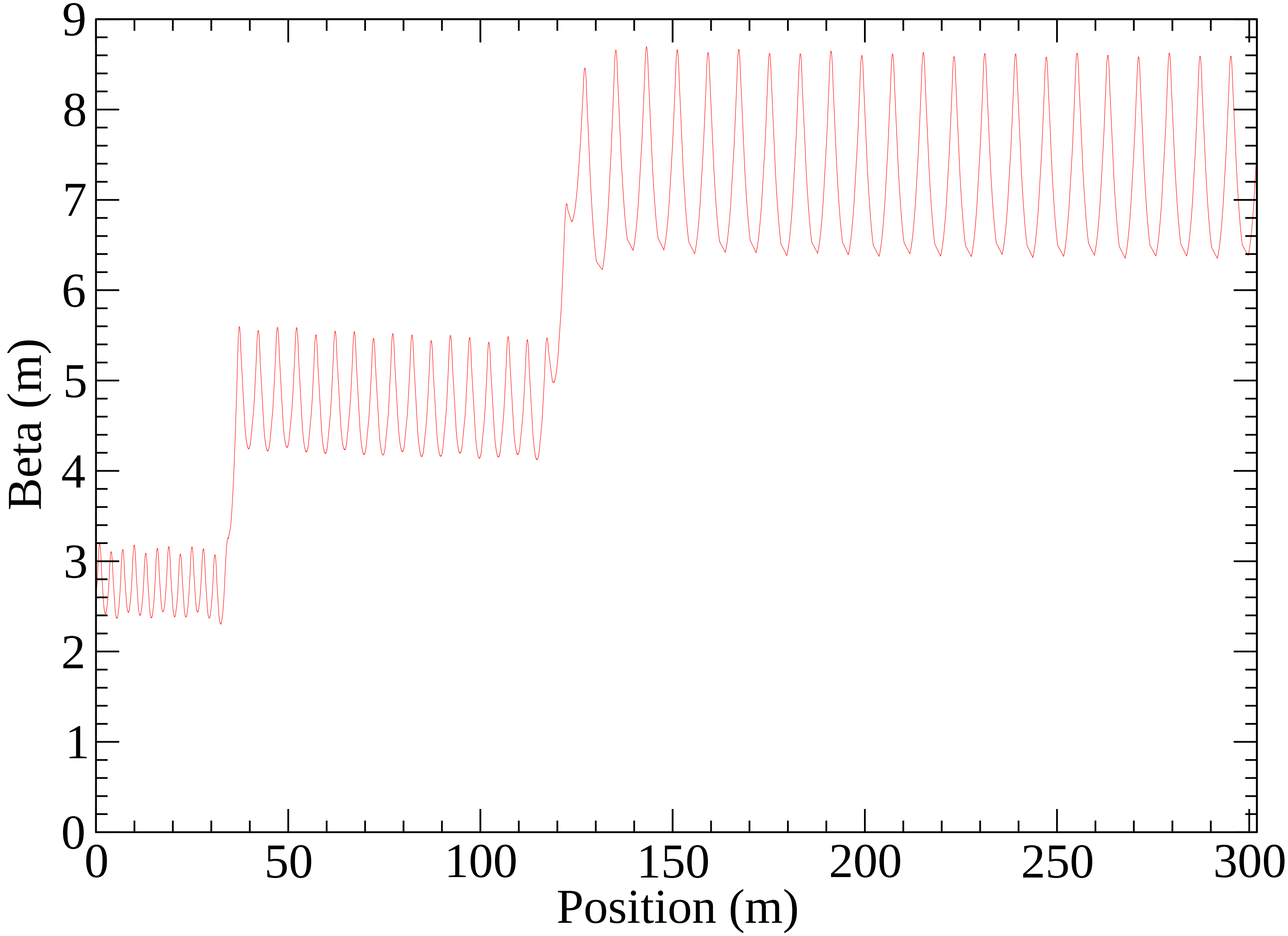
Longitudinal Acceptance

- Don't quite get 150 mm acceptance: close at start, not quite at end
  - Difference from Alex: lower effective gradient at beginning (transit time factor)
- Possible cures
  - Start further off crest: but already pretty far off crest (73°)!
  - May be caused by distortion at end on crest!
    - ⋆ Can check this: take snapshots at different points
    - ⋆ If so, want to narrow distribution more: can we modify RF phase profile?
  - Tighter lattice: shorter solenoids?
- Average energy is below reference particle energy (about 25 MeV)

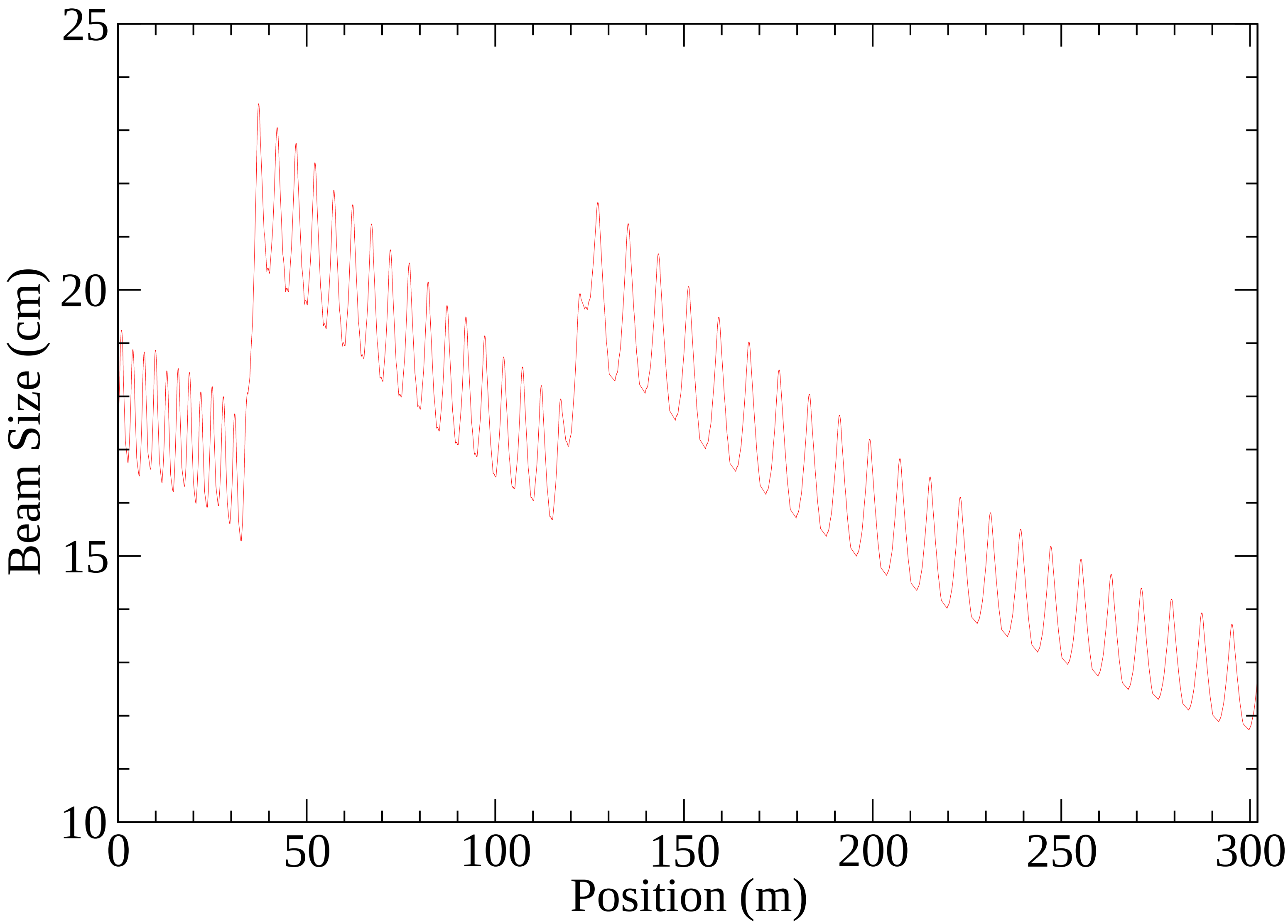
Transverse Linear Lattice

- Send small amplitude particle through ICOOL to compute transfer matrix
- Compute beta functions, beam sizes at acceptance
  - Beginning of second stage is a bit large, but not too bad
  - Plenty of room at beginning of first stage
    - Could start at lower energies
    - Longitudinal acceptance is the issue

Beta Functions



Beam Size



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Continued Work

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- Fix the longitudinal acceptance
  - Or demonstrate that we can live with it (e.g., subsequent systems transmit the distorted phase space)
- Check for emittance growth (tracking done, analysis not)
- Use more realistic solenoid model: nonlinearities
- Model remaining components (I have the dogbone linac. . .)

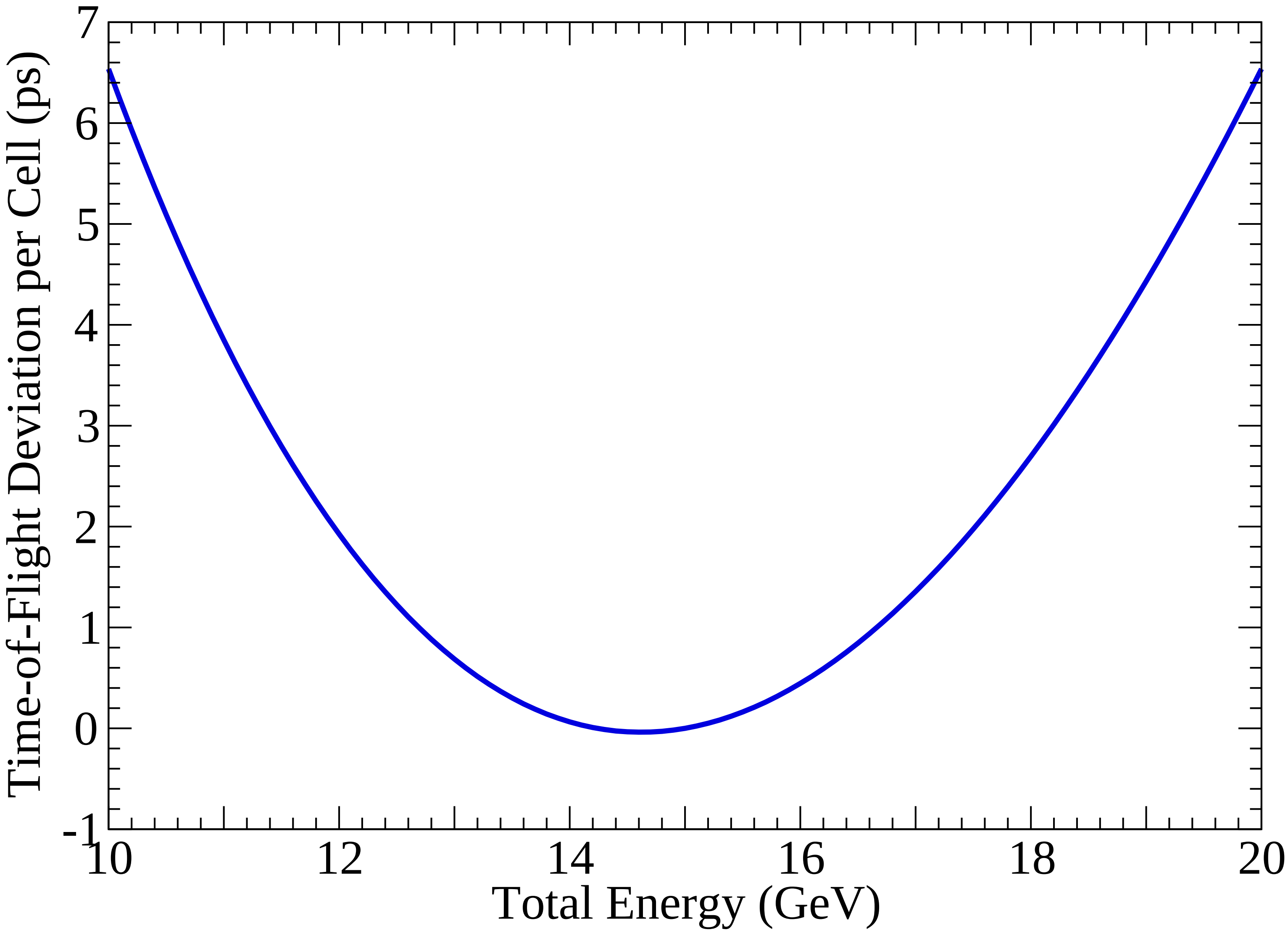
Outline: FFAG Designs

- Review of optimization process
- Review of previous results
- Updated Cost Model
- Characteristics of optimal lattices
- Minimum cost rings
- Decay cost
- Parametric dependencies of lattices
- New lattices
- Remaining work
- Conclusions

Review of Optimization Process

- Muon FFAG lattices consist of several identical cells of a particular type (doublet, FDF triplet, FODO)
- Assume 201.25 MHz RF
- A drift of at least 2 m is specified for the RF cavity
  - Purpose: keep field on superconducting cavities below 0.1 T
- Leave 0.5 m of space between magnets in doublet/triplet
- Time-of-flight vs. energy is parabolic-like; set height of parabola at min and max energy to be same
- For longitudinal acceptance, constrain $a = V/(\omega\Delta T\Delta E)$
  - ΔT is height of parabola (one turn), V is total voltage installed
  - Value of a depends on energy range, empirically chosen, increases with decreasing energy
- Factor of 2 in energy: 2.5–5 GeV, 5–10 GeV, 10–20 GeV

Time-of-Flight vs. Energy



Review of Previous Results of Optimization

- Doublet lattice is most cost effective
  - Triplet lattice has lowest voltage requirement, but
  - Three magnets per cell drives up magnet cost
  - Difference FD $\longrightarrow$ FDF $\longrightarrow$ FODO is around 5% each
- Cost per GeV of acceleration increases rapidly as energy decreases
  - 2.5–5 GeV of questionable cost value for muon acceleration

Updated Cost Model (Palmer)

- Compared to previous model
  - Cost at zero field for fixed magnet size does not go to zero
  - A new symmetry factor (quad/dipole/combined function) is used
    - Proportional to amount of coil needed
    - Factor is identical for dipoles and quadrupoles
    - Factor is less than 1 for combined function
- Basic formula: product of 4 factors

$$f_B(\hat{B}) f_G(\hat{R}, L) f_S(B_-/B_+) f_N(n)$$

  - f_B: dependence on field
  - f_G: geometric dependence: magnet length L
  - f_S: symmetry dependence
  - f_n: dependence on number of magnets being made n

- For linear midplane field profile $B_y = B_0 + B_1 x$,

$$B_{\pm} = |B_0| \pm |B_1| \, k_R R$$

- Peak field and larger radius it requires

$$\hat{B} = B_+ + |B_1| \, k_C B_+ \qquad \hat{R} = k_R R + k_M \hat{B}$$

- The factors

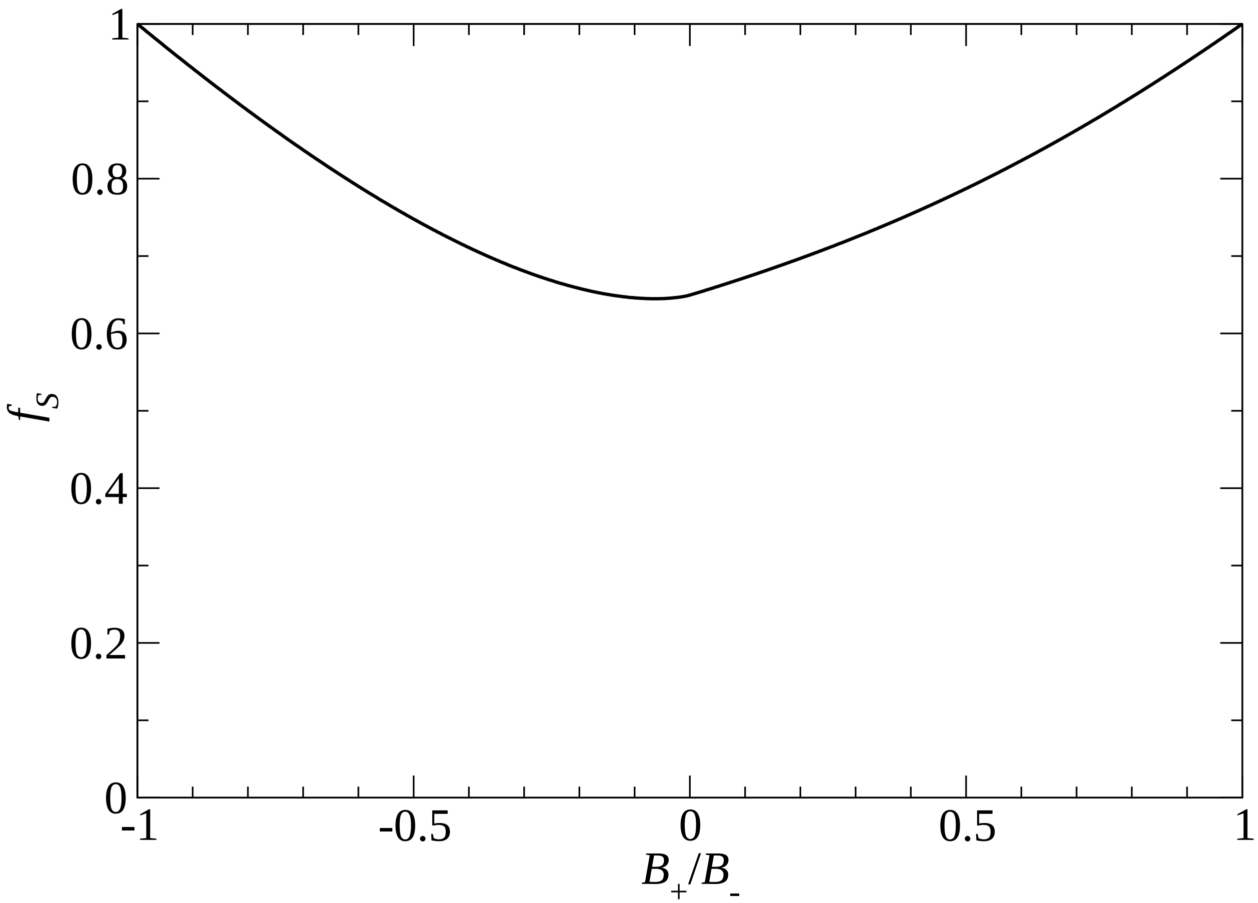
$$f_B(\hat{B}) = C_0 + C_1 \hat{B}^{k_B} \qquad f_G(\hat{R}, L) = \hat{R}(L + k_G \hat{R})$$

$$D = (1 + B_-/B_+)/2 \qquad Q = (1 - B_-/B_+)/2 = 1 - D$$

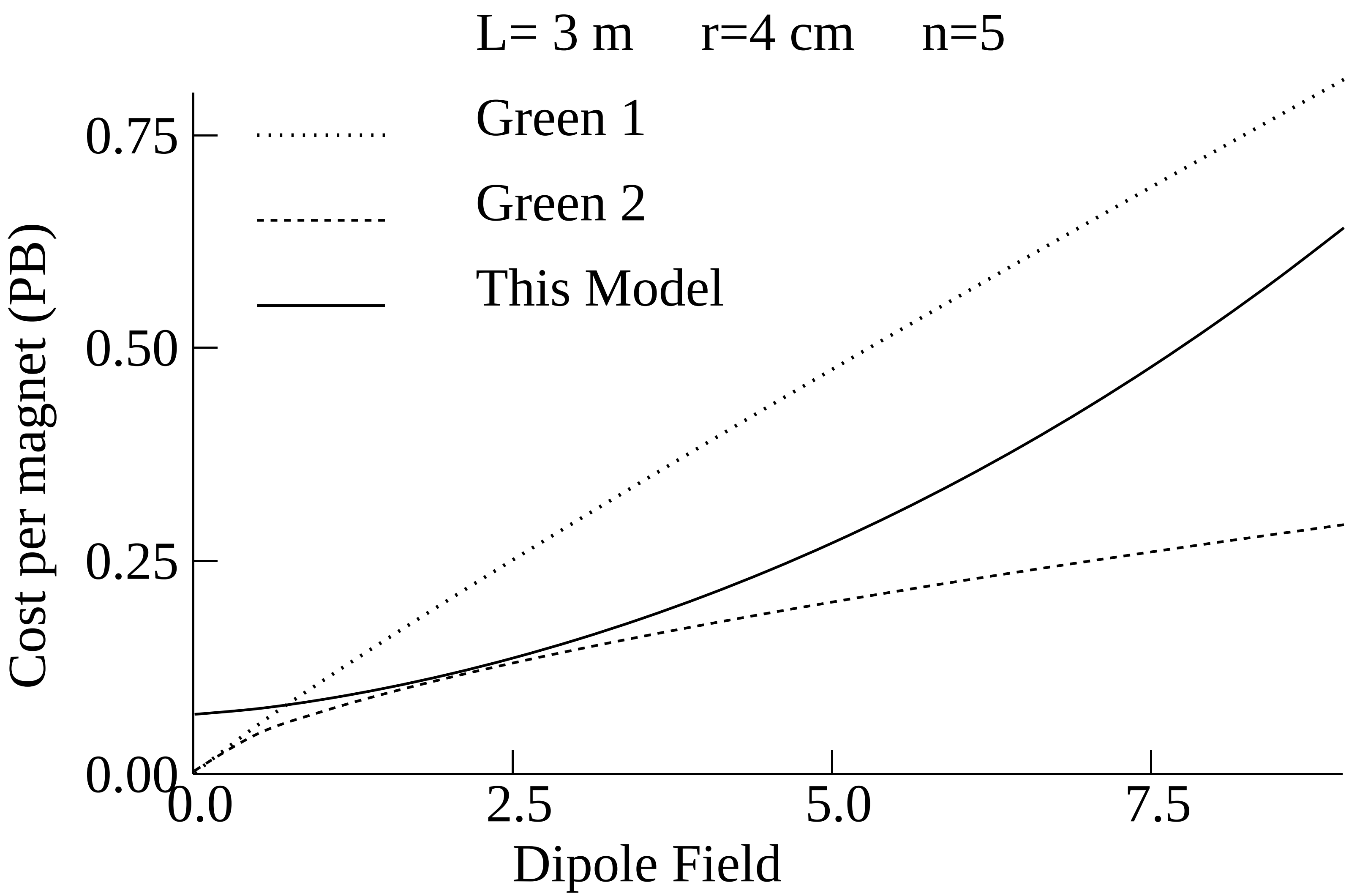
$$f_S(B_-/B_+) = \frac{\int_0^\pi |D \cos\theta + Q \cos 2\theta| \, d\theta}{\int_0^\pi |\cos\theta| \, d\theta} \qquad f_N(n) = (n_0/n)^{k_N}$$

Updated Cost Model (cont.)

k_R	1.3
k_C	2.47 mm/T
k_M	2 mm/T
C_0	0.101 PB/m^2
C_1	16.78 mPB/T$^{1.5}$/m^2
k_B	1.5
k_G	36
k_N	1/3
n_0	300



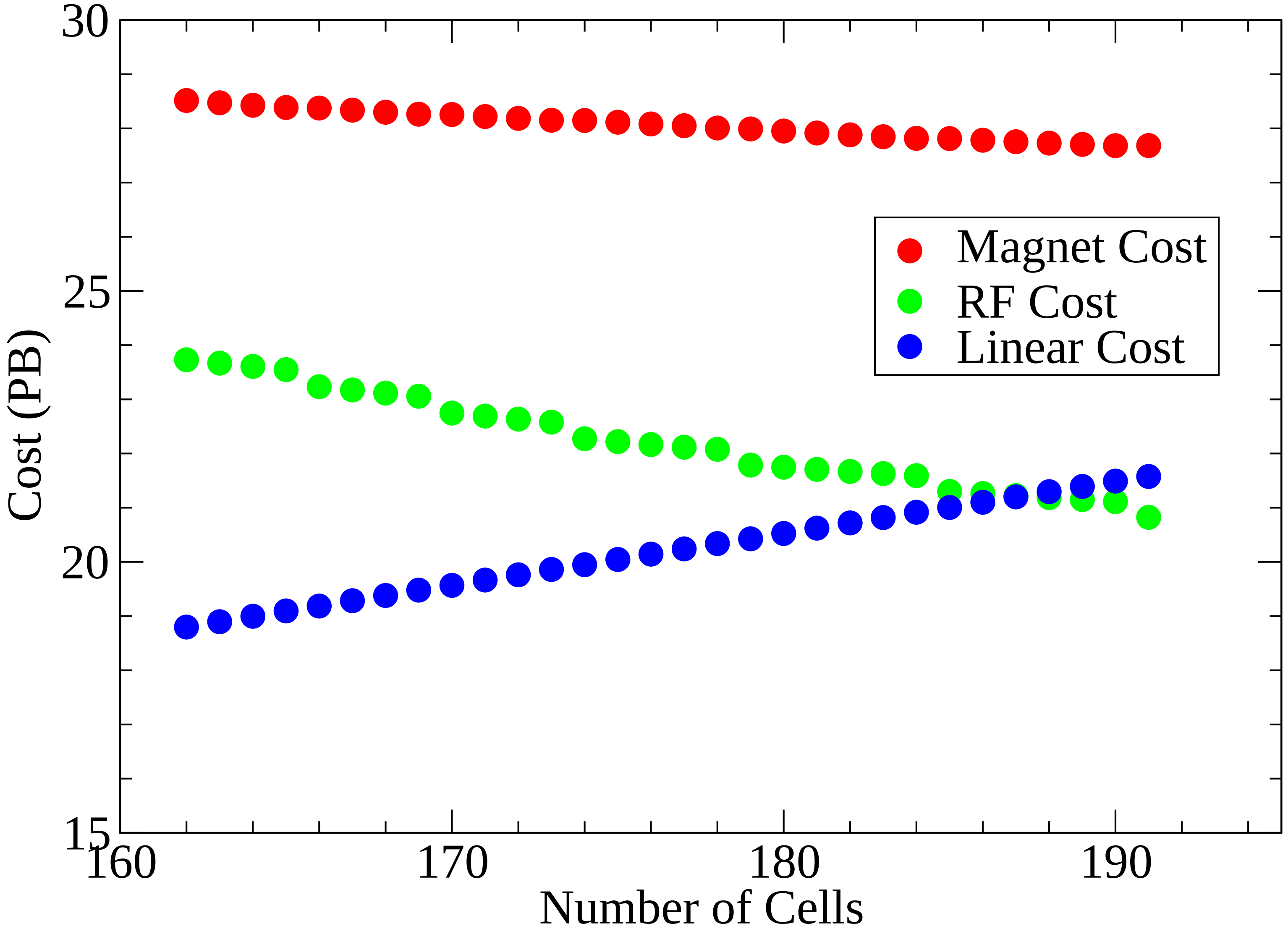
Updated Cost Model (cont.)



Characteristics of Optimal Lattices

- For modest lengths, lattice (magnet+linear) cost decreases with increasing circumference
  - Reduced dispersion reduces aperture requirement
  - Remarkably, this cost reduction is goes down more quickly than inversely in the number of cells
  - At some point, this stops as the nonzero transverse beam size stops the decrease in the aperture
  - The minimum-cost solution does not have every cell filled with RF!

Costs vs. Number of Cells



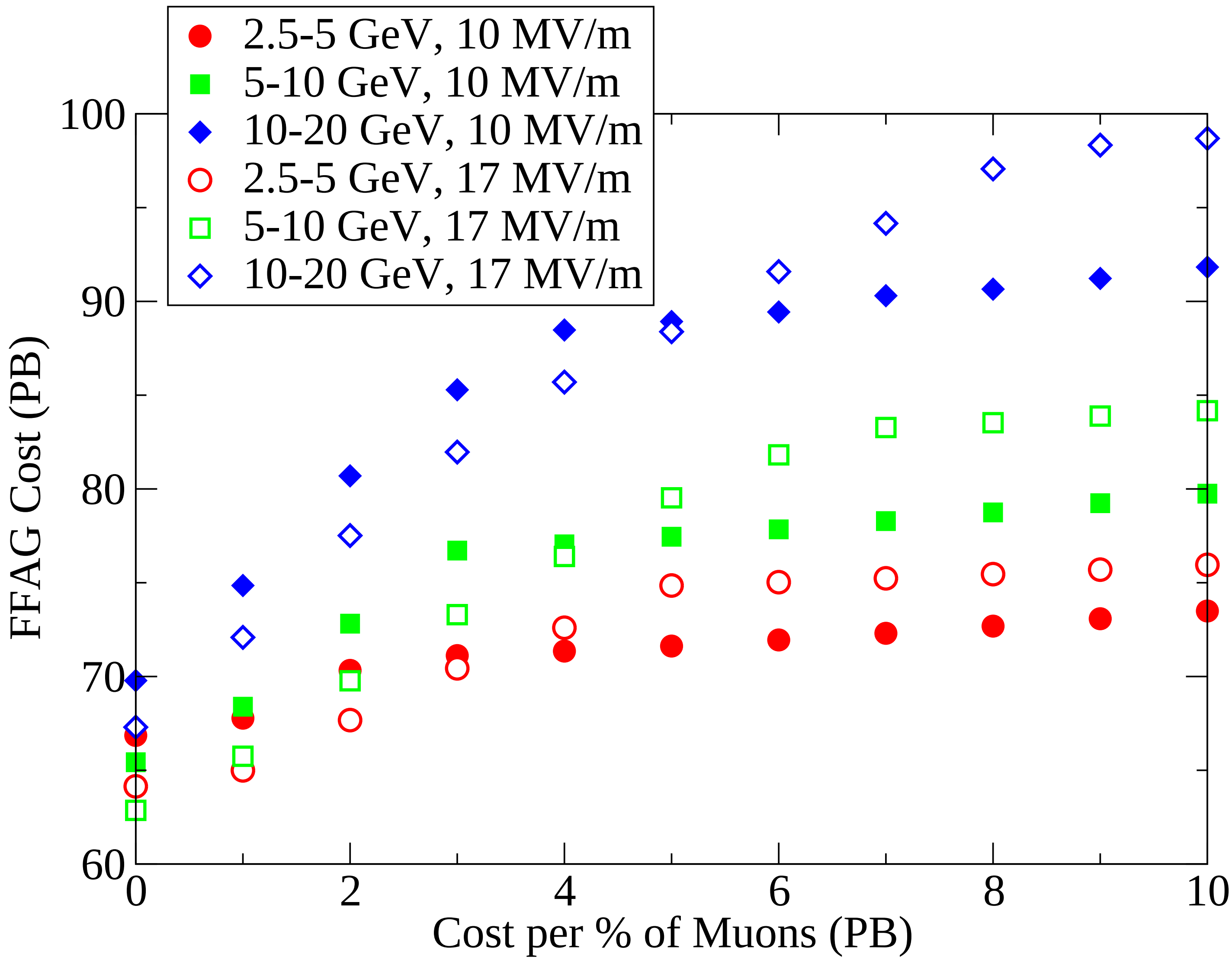
Decay Cost

- The minimum cost rings are extremely long
  - Decays are unacceptably high
- Need to incorporate tradeoff between decays and cost of acceleration into optimization
  - Simplest thinking: can always make detector larger to make up for lost particles
  - Multiply detector cost by fractional loss
  - Over-simplifies things (e.g., as detector gets larger, fractional increase costs more)
  - Baseline: detector costs 500 PB

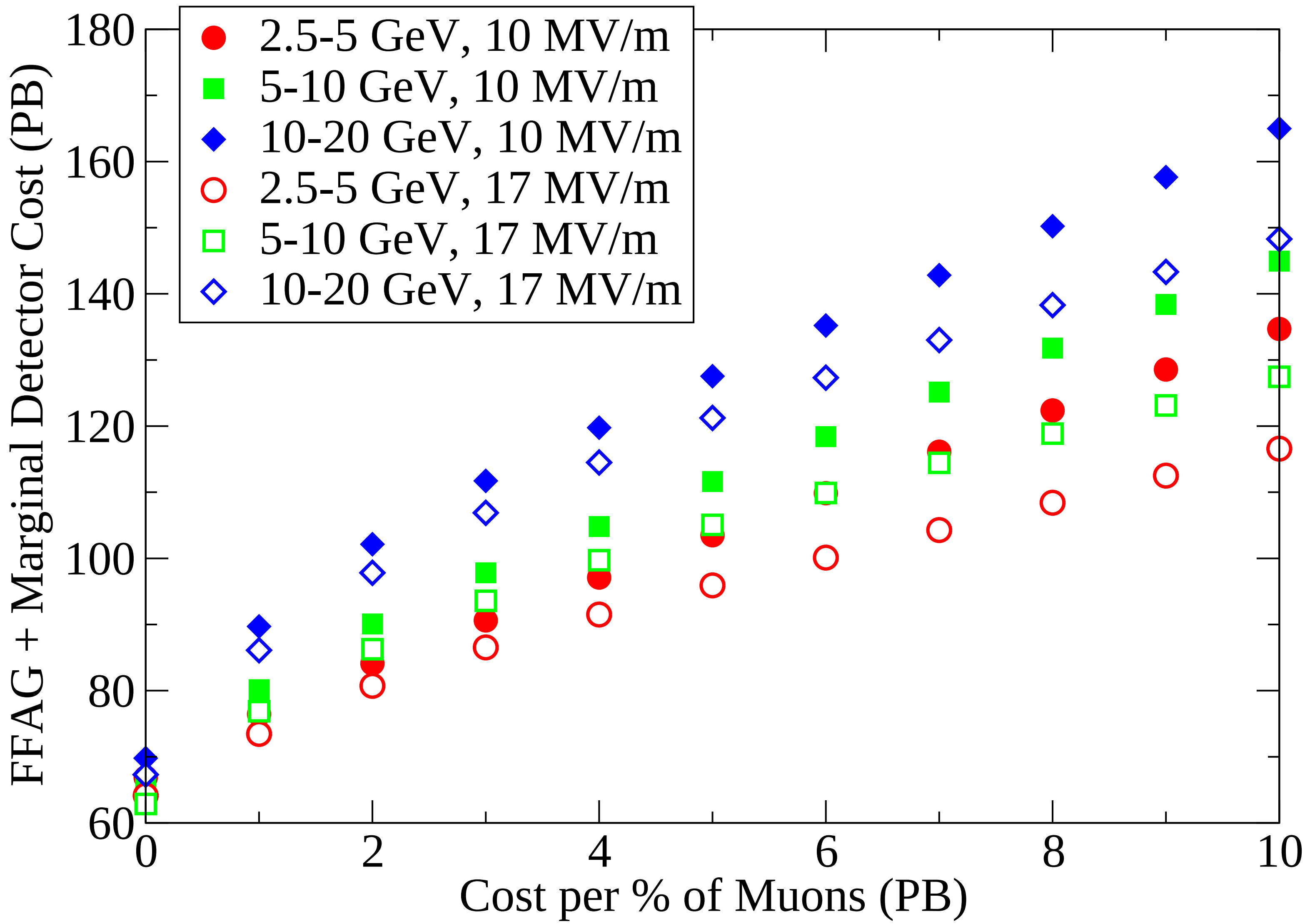
Parametric Dependencies

- Cost vs. decay cost
  - For low decay cost, ring is partially filled
  - As decay cost increases, ring optimized to reduce decay
    - More RF
    - Ring shortens
  - Once ring is filled, can't increase RF or shorten ring easily
    - Ring shortens slightly: magnets shorter, higher field
    - To get little gain, large increase in cost
    - Detector cost increases more rapidly at this point
  - Higher gradient, can go longer before ring is filled
  - Total cost steadily increases with increasing decay cost

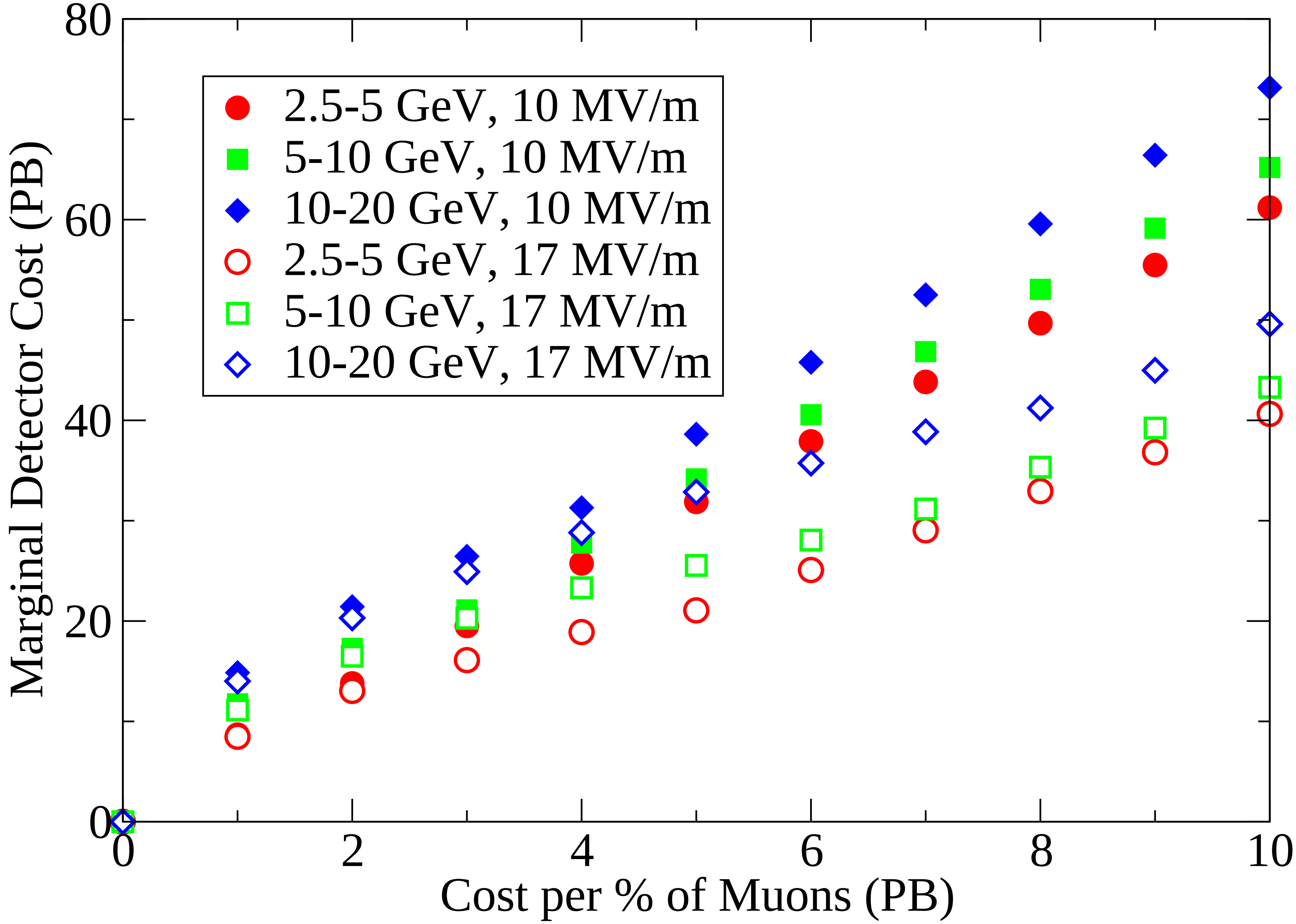
FFAG Cost vs. Decay Cost



Total Cost vs. Decay Cost



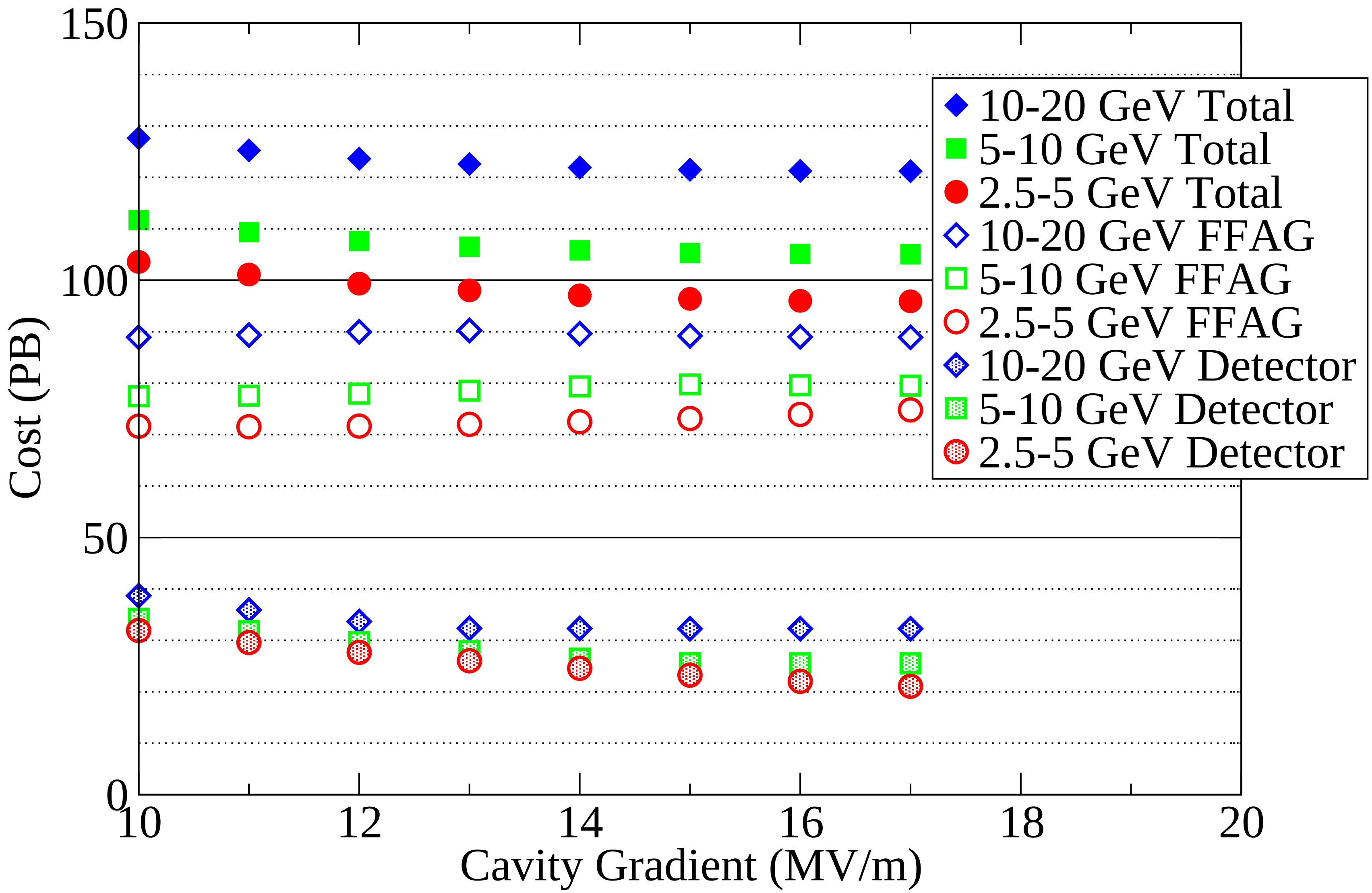
Marginal Detector Cost vs. Decay Cost



Cost vs. Gradient

- Use 5 PB/% for the muon cost
- Relatively weak dependency: higher gradient may not be worth it
  - Assumed structure costs independent of gradient
    - Might need better surface
    - Tougher requirements on input couplers
  - Higher cryo costs
- FFAG cost increases with increasing gradient for low gradients
  - Total cost decreases since detector cost decreases
  - Ring is filled
    - Total voltage increases faster than cost per voltage
    - Ring circumference decreases, increasing ring cost
- Higher gradients, can partially fill ring
  - Roughly same voltage and circumference
  - Fewer cavities

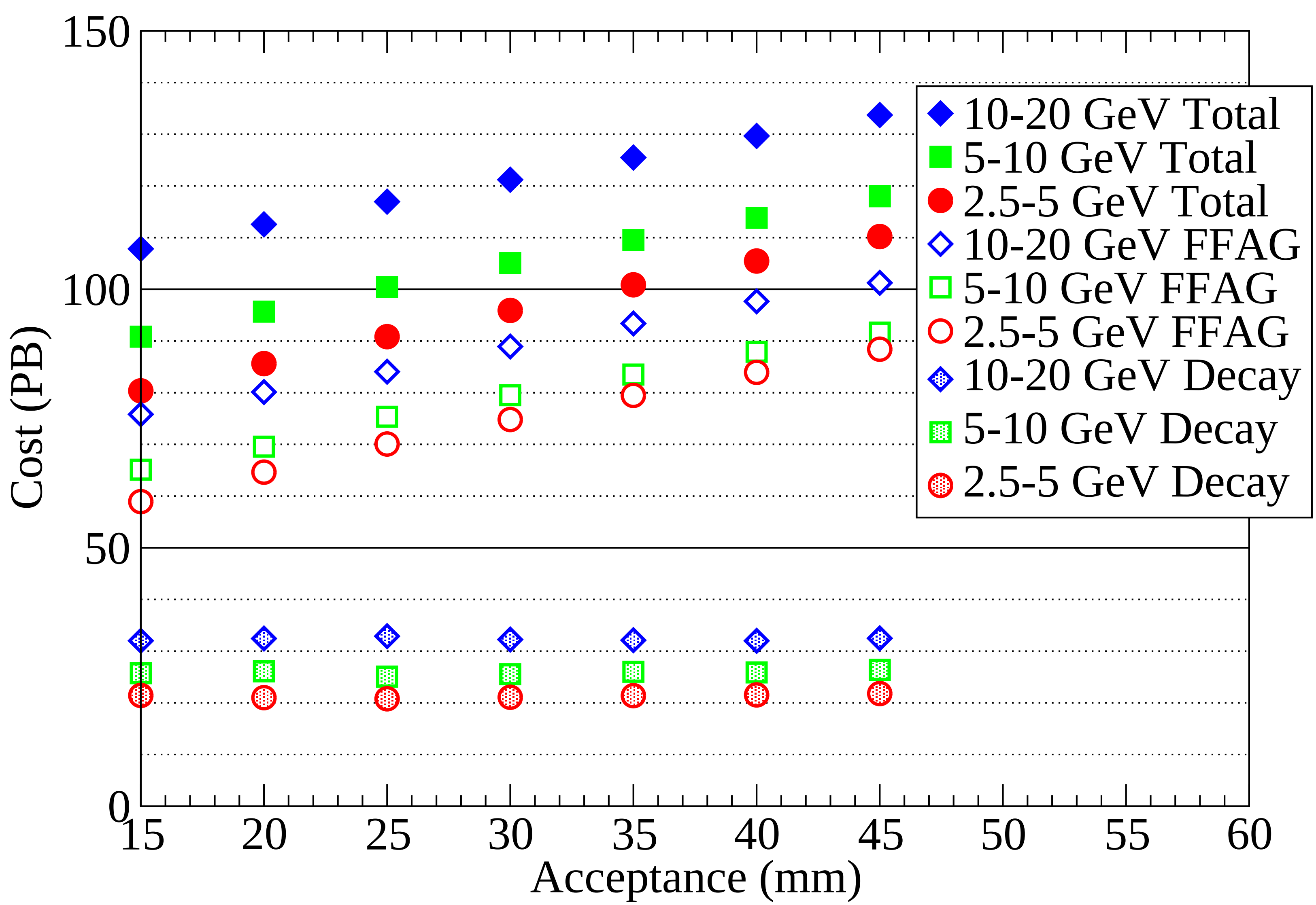
Cost vs. Gradient



Cost vs. Acceptance

- Strong dependence of cost on acceptance
- Primarily caused by increased magnet cost
  - Primarily coming from increased size (length and aperture)
  - Not really coming from increased fields

Cost vs. Acceptance



Another Mind-Numbing Lattice Table

Minimum total energy (GeV)	2.5	5	10
Maximum total energy (GeV)	5	10	20
$V/(\omega\Delta T\Delta E)$	1/6	1/8	1/12
No. of cells	50	65	82
D length (cm)	63	77	97
D radius (cm)	13.4	10.0	7.4
D pole tip field (T)	4.5	5.7	7.1
F length (cm)	96	113	140
F radius (cm)	21.2	16.3	13.1
F pole tip field (T)	2.7	3.5	4.3
No. of cavities	42	49	56
RF voltage (MV)	534	620	703
Turns	4.7	8.2	15.0
Circumference (m)	204	286	399
Decay (%)	4.2	5.1	6.5
Magnet cost (PB)	39.4	37.2	39.1
RF cost (PB)	30.3	35.2	39.9
Linear cost (PB)	5.1	7.1	10.0
Total cost (PB)	74.8	79.5	88.9
Cost per GeV (PB/GeV)	29.9	15.9	8.9

- Decay cost: 5 PB/%
- Acceptance 30 mm
- Choose 17 MV/m: Study II baseline
- Pole tip fields are higher than previously
- 2.5–5 GeV is borderline

Remaining Work for Optimization

- Choice of $V/(\omega\Delta T\Delta E)$ still empirical
  - I have a method of doing this, just haven't finished the calculations
- Work on choice of cavity drift length and inter-magnet drift
  - Let it depend on the magnet fields/apertures? How?
- Choice of aperture: should be coupled to cooling design
  - Can compute cooling cost vs. aperture when muon cost is included
  - Cooling cost decreases with increasing aperture
  - Add cooling cost and acceleration cost vs. aperture
  - Presumably there is an optimum aperture

Conclusions: FFAG Optimization

- I am using an improved cost model from Palmer
- An earlier notion that magnet costs increase with increasing number of cells was wrong. This has been addressed by including decay costs in the model.
- I have a set of lattices which are optimal to my current understanding
- I can produce "optimal" lattices at will for given constraints
- There are always improvements to be made…